on the Moon's longitude at first and third quarters. It will therefore enter with its full effect into the parallactic inequality, but will be eliminated from the mean longitude.

2. Since all the observations of altitude must be made on the Moon's lower limb, the effect of error of semi-diameter will be the same as that of the altitude.

3. The errors in azimuth will practically have the same effect as those in right ascension, both as regards the limb and the errors which are a function of the azimuth itself.

For these reasons we must not expect the two methods to agree. Until we free the altazimuth observations from the source of error in question, it seems to me not safe to rely upon them for the parallactic inequality. To do this requires a more careful study of the instrument and the method of using it than I am aware of having been made.

Washington: 1904 *May* 11.

Methods of Correcting Moon's Tabular Longitude.

By P. H. Cowell.

In previous papers I have given a list of about a dozen corrections applied to Hansen's tabular longitudes from 1847·0 onwards. I now give a list of corrections to Airy's tabular longitudes, 1750–1851, that have already been entered, though the additions have not as yet been completed.

Terms whose arguments are multiples of the same angle are combined in one table, and in the following list are given under a single reference number; for example, the first table applies the correction

$$-2''\cdot58 \sin D - 0''\cdot20 \sin 2D - 0''\cdot37 \sin 3D - 0''\cdot46 \sin 4D$$

all at once.

The only exception to this is the terms in g' and $2g'$, where the secular variation of the former made it necessary to treat the latter separately.

The first list contains terms of short period; the second list terms of longer period:

Ref. No.	Argument.	Coefficient of Correction Applied.	Coefficient used by Airy *plus* Coefficient of Correction.	Remarks.
1	D	$-2''\cdot58$	$+ \; 124''\cdot68$	Parallactic Inequality
1	2D	$-0\cdot20$	$- \; 2370\cdot10$	Variation
1	3D	$-0\cdot37$	$+ \; 0\cdot53$	
1	4D	$-0\cdot46$	$+ \; 13\cdot94$	

Ref. No.	Argument.	Coefficient of Correction Applied.	Coefficient used by Airy *plus* Coefficient of Correction.	Remarks.
2	$-2g-g'$	$+0''{\cdot}38$	$+\ 7''{\cdot}68$	
3	$-g-g'$	$-1{\cdot}20$	$+\ 109{\cdot}90$	
4	$g-2g'$	$+0{\cdot}49$	$+\ 2{\cdot}59$	
5	$g+2\omega-2\omega'$	$-0{\cdot}45$	$-\ 2{\cdot}45$	
6	$2g-g'+2\omega-2\omega'$	$-0{\cdot}88$	$24{\cdot}48$	
7	$-g-2g'+2\omega-2\omega'$	$+0{\cdot}44$	$+\ 13{\cdot}24$	
8	$g-3g'+2\omega-2\omega'$	$-3{\cdot}30$	$+\ 206{\cdot}40$	
9	$2g-3g'+2\omega-2\omega'$	$-0{\cdot}38$	$+\ 165{\cdot}52$	
10	$3g-3g'+2\omega-2\omega'$	$+0{\cdot}63$	$+\ 14{\cdot}63$	
11	$4g-3g'+2\omega-2\omega'$	$+0{\cdot}56$	$+\ 1{\cdot}16$	
12	$3g-4g'+2\omega-2\omega'$	$+0{\cdot}44$	$+\ 0{\cdot}74$	
13	$2g+g'+2\omega$	$+0{\cdot}42$	$+\ 0{\cdot}42$	Term omitted by Airy.
14	$g+2\omega$	$-2{\cdot}36$	$-\ 39{\cdot}56$	
15	$2g+2\omega$	$-0{\cdot}62$	$-\ 411{\cdot}62$	
16	$-g+3g'+2\omega'$	$+0{\cdot}38$	$+\ 0{\cdot}38$	Term omitted by Airy.
17	$2g+2g'+2\omega'$	$+0{\cdot}35$	$+\ 0{\cdot}55$	
18	$g+\omega-\omega'$	$+0{\cdot}70$	$+\ 17{\cdot}90$	
19	$-g-g'+\omega-\omega'$	$-0{\cdot}78$	$-\ 1{\cdot}58$	
20	$g-3g'+3\omega-3\omega'$	$-0{\cdot}61$	$-\ 1{\cdot}21$	
21	$g-4g'+4\omega-4\omega'$	$+0{\cdot}60$	$+\ 1{\cdot}10$	
22	$3g-4g'+4\omega-4\omega'$	$+0{\cdot}46$	$+\ 38{\cdot}46$	
23	$5g-4g'+4\omega-4\omega'$	$+1{\cdot}03$	$+\ 1{\cdot}93$	
24	$2g-5g'+4\omega-4\omega'$	$+1{\cdot}55$	$+\ 2{\cdot}75$	
25	$3g-5g'+4\omega-4\omega'$	$+0{\cdot}58$	$+\ 4{\cdot}38$	
26	$4g-5g'+4\omega-4\omega'$	$+0{\cdot}58$	$+\ 1{\cdot}78$	
27	$2g-2g'+4\omega-2\omega'$	$+0{\cdot}36$	$-\ 0{\cdot}54$	
28	$4g-2g'+4\omega-2\omega'$	$-2{\cdot}34$	$-\ 5{\cdot}74$	
29	$5g-2g'+4\omega-2\omega'$	$-0{\cdot}40$	$-\ 1{\cdot}00$	
30	$3g-3g'+4\omega-2\omega'$	$-0{\cdot}43$	$-\ 0{\cdot}43$	Term omitted by Airy.
31	$4g-6g'+6\omega-6\omega'$	$+0{\cdot}54$	$+\ 0{\cdot}54$	,, ,,
32	$2g-5g'+2\omega-4\omega'$	$-0{\cdot}40$	$0{\cdot}00$	Airy's wrong term.
33	$g+2\varpi-2\mathrm{J}$	$-0{\cdot}893$	$-\ 0{\cdot}893$	Jupiter term.
34	$g+2\varpi-3\mathrm{J}+7^\circ$	$-0{\cdot}316$	$+\ 0{\cdot}316$	,, ,,
35	g	$-2{\cdot}10$	$+22639{\cdot}50$	Anomaly.
35	$2g$	$-0{\cdot}46$	$+\ 769{\cdot}04$	
35	$3g$	$-0{\cdot}58$	$+\ 36{\cdot}12$	
36	$2\mathrm{D}-g$	$+0{\cdot}738$	$+\ 4586{\cdot}338$	Evection.
36	$4\mathrm{D}-2g$	$-3{\cdot}75$	$+\ 30{\cdot}75$	

Ref. No.	Argument.	Coefficient of Correction Applied.	Coefficient used by Airy *plus* Coefficient of Correction.	Remarks.
I.	$-g'+2\omega-2\omega'$	$+3''.80$	$+\ 2''.40$	
II.	$-g'+\omega-\omega'$	-0.60	$-\ 18.60$	
II.	$-2g'+2\omega-2\omega'$	-0.65	$+211.75$	
III.	$-3g'+2\omega-2\omega'$	$+0.86$	$+\ 8.66$	
IV.	$-g'$	$\begin{cases} +1.03 \\ -1.63\,T \end{cases}$	$+669.63$ $-\ 1.63\,T$	Annual equation. T in centuries from 1800
V.	$-2g'$	-0.40	$+\ 7.50$	
VI.	$+2g'+2\omega'$	-0.30	$-\ 55.20$	
VII.	V − E	$+0.30$	$-\ 0.80$	Venus term
VIII.	$2V-3E+85^\circ$	-0.30	$-\ 0.30$	" "

Subsequently a term in cos g will probably be added to the above list; but I propose carrying out analyses for g and D without applying any further short period corrections.

I now proceed to explain the method employed.

The transits of the Moon, whether observed or not, are numbered consecutively, counting from 1750 September $13^d\ 10^h$ as No. 1; for convenience they are then divided into forties; and the 29th and 30th batch of forty are shown in the columns headed [29] and [30] respectively. The blank in the first line of column [29] informs us that the Moon was not observed at the corresponding transit which is clearly 28×40 lunar days later than column [1], line 1, which corresponds to 1750 September $13^d\ 10^h$. If anyone is interested in expressing this date in terms of the civil calendar, it is easy to do so, by taking the lunar day as 1·03505 of a solar day. The entry −50 in the second line informs us that Airy's tabular longitude of the Moon exceeded the observed longitude at the corresponding transit by −50 units of one-tenth of a second of arc. This is the result of pure copying from Airy's Greenwich Lunar Reductions, with the following modifications only: (i) the second decimal of a second is omitted; (ii) the decimal point is omitted; (iii) the sign is changed, as Airy gives observed *minus* tabular; (iv) means are taken when Airy gives more than a single result, that is to say, when both first and second limbs have been observed. The third line shows that the corresponding transit was not observed; then follow three observations, and then seven unobserved transits (during which it is clear that new moon must have fallen) and so on.

Table 7.

	7	[29]	7	[30]	1	2
1	...	...	...	...	A + 1	− 4
2	− 1	− 50	...	...	+ 2	− 4
3	=	...	—	—	+ 3	− 4
4	+ 1	− 20	− 4	− 72	+ 4	− 3
5	+ 2	− 46	− 4	− 74	+ 4	− 2
6	+ 3	− 38	...	...	+ 4	− 1
7	...	...	− 3	− 28	+ 4	0
8	...	...	− 2	− 12	+ 4	+ 1
9	...	...	...	...	+ 3	+ 3
10	...	...	0 =	+ 63	+ 2	+ 3
11	...	...	+ 1	+ 78	+ 1	+ 4
12	...	...	+ 2	+ 71	− 1	+ 4
13	...	...	+ 3	− 2	− 2	+ 4
14	+ 1	− 12	+ 4	− 27	− 3	+ 4
15	0	− 29	...	...	− 4	+ 4
16	...	...	+ 4	− 31	− 4	+ 3
17	− 2	− 45	...	...	− 4	+ 2
18	− 3	+ 19	...	...	− 4	+ 1
19	...	...	...	...	− 4	− 1
20	—	...	...	...	− 3	− 2
21	− 4	− 13	...	...	− 3	− 3
22	− 4	+ 64	0	0	− 1	− 4
23	− 4	+ 87	− 1	+ 31	0	− 4
24	− 3	+ 70	...	...	B + 1	− 4
25	...	...	...	...	+ 2	− 4
26	...	...	...	...	+ 3	− 4
27	...	...	...	...	+ 4	− 3
28	...	...	...	...	+ 4	− 2
29	...	...	...	...	+ 4	− 1
30	...	...	...	...	+ 4	0
31	...	...	− 3	− 41	+ 4	
32	+ 4	− 63	...	...	+ 3	
33	...	...	...	...	+ 2	
34	...	...	0	− 83	+ 1	
35	...	...	+ 1	− 89	0	
36	...	...	...	...	− 1	
37	...	...	...	...	− 2	
38	...	...	+ 4	− 21	− 3	
39	...	...	+ 4	+ 26	− 4	
40	...	...	+ 4	+ 46	− 4	

Precepts

	c	l
B	29	4
B	30	11
B	31	18
B	32	25
A	33	32
B	34	39

The computing form annexed is to be considered as consisting of three, or rather four, separate pieces of paper. The first piece contains column [29] as its extreme right-hand column, to the left lies the various corrections of which correction 7 is alone shown; the second piece shows column [30] and correction 7 in a similar manner; the third and fourth pieces are mere strips which may be referred to as table 7, columns 1 and 2, respectively.

The following is the process of entering the corrections: the last three entries of table 7, column 2, are held in juxtaposition to the first three lines of column [29]; that this is the right position is inferred from the fact that the preceding entry of table 7 has been previously found to apply to column [28] since 40. Holding the slip in his left hand and to the left of column [29], the computer then enters the correction whenever the transit has been observed, *e.g.* he does not copy −2 and 0 in the first and third lines, but he does copy −1 in the second line. Having now come to the end of table 7 he draws a double line as shown. When table 7 comes to an end it is always proper to begin again, either at the beginning (the first entry is marked A) or at the 24th entry, which is marked B. To ascertain at which of these two points to begin in the present instance, it is necessary to refer to the Precepts, an extract from which is annexed, stating that at column [29], line 4, the proper entry is B. This precept serves the further purpose of preventing mistakes, for, supposing for example that in turning over from column [28] to column [29] the computer slipped one line, he would discover the mistake on finding that the precept did not appear to come at the right line; he would then go back to the preceding precept, which has presumably come right, and make the necessary alterations, and the mistake is prevented from continuing undiscovered throughout an indefinite number of columns.

The plan of the table is obvious. The entries are the products of $-4{\cdot}4$ by the sines of angles in an arithmetical progression, the common difference being $15^\circ{\cdot}322690$, the movement of the argument in one lunar day. The corrections thus applied correspond to mean lunar noons, but no serious error is committed by applying them at apparent lunar noons.

To save a little labour in constructing the table the argument of the middle entry (in the present case the 35th) is taken as exactly zero. The second half of the table is then constructed, and the first half follows from it by a change of sign and reversing the order of the entries.

I give below the full calculation for the table:

Entry.	Argument.	Entry.	Argument.
35	$0^\circ{\cdot}00$	58	$0^\circ - 7^\circ{\cdot}35$
46	$180^\circ - 11^\circ{\cdot}34$	59	$0^\circ + 7^\circ{\cdot}98$
47	$180^\circ + 3^\circ{\cdot}99$	70	$180^\circ - 3^\circ{\cdot}36$

n	$\log\left\{\frac{2n-1}{8 \cdot 8}\right\}$	Anti-log. sine.	Residual after subtracting multiples of 15·332690.
		°	°
1	0555	6·52	6·52
2	5326	19·93	4·60
3	7545	34·63	3·96
4	9006	52·70	6·70
		90·00	13·33

The above little computation is sufficiently explained by its headings; the last column informs us (second line for example) that $4 \cdot 4 \times \sin(15^\circ \cdot 332690 + \theta) = 1$ if θ be less than 4·60 and $= 2$ if θ be greater than 4·60; the last line informs us that $5 \times 15^\circ \cdot 332690 + \theta$ exceeds 90° if θ exceeds 13°·33. Table 7 is now constructed six entries at a time by merely referring to the last column of the above computation; for instance, the entries Nos. 46–41 in the reverse order are seen to be -1, -2, -3, -4, -4, -4, by merely comparing 11°·34 (see value of argument for entry No. 46) with the angles that stand in the above column.

I come now to the formation of the precepts. The movement of the argument in seventy lunar days is $3 \times 360^\circ - 6^\circ \cdot 7117$; in forty-seven lunar days is $2 \times 360^\circ + 0^\circ \cdot 63643$. From this it may be seen that to "Return to A" adds $-6^\circ \cdot 7117$ to the excess of the true argument (*i.e.* the true value of the argument at the nearest mean lunar noon) over the argument of the table; while to "Return to B" adds $+0^\circ \cdot 63643$ to the same excess. Consequently it follows that the difference between the true argument and the argument of table can always be kept less $\frac{1}{2}\{6^\circ \cdot 7117 + 0^\circ \cdot 63643\}$ or 3°·67, and the maximum error committed will be less than $0'' \cdot 44 \times$ circular measure of 3°·67, or less than 0″·03. Had it been considered desirable the error might have been confined to narrower limits by using a larger table. The numbers 47 and 70 are readily found by turning $\frac{15^\circ \cdot 332690}{360^\circ}$ into a continued fraction, the first two convergents are $\frac{1}{23}$ and $\frac{2}{47}$; of the wo numbers 47 and 70, therefore, the former is the denominator of the last convergent, and the latter the sum of the denominators of the two last convergents.

It will be seen that the method is perfectly general; it can be applied to corrections of any size and to any degree of accuracy at the cost of enlarging the table. If the coefficient of the correction is N times the maximum permissible error, the table must consist of 2πN entries at least. If, however, the table had been required for an ephemeris instead of for a series of discontinuous dates, it would be reasonable to throw such operations as changing signs and copying backwards upon the computer, and the size of the table could be divided by 4, making $\frac{\pi}{2}$N entries at the least. The use of the table involves mere copying without interpolation, and it is clear that any table professing to tabulate a quantity

that may take all values from 1 to N without interpolation correctly to a single unit, must contain N entries. The coefficient of efficiency of the method may therefore be taken as $\frac{2}{\pi}$ (perfection being taken as unity).

This slight loss of efficiency is clearly not a high price to pay for reducing the entering to pure copying without interpolation, and abolishing the calculation of the arguments for the individual observations.

As I pointed out in the *Observatory* for 1903 July, in the case of large corrections interpolations may be grafted on. Suppose, for instance, it were required to tabulate $22639''\cdot50 \sin g$ correctly to $0''\cdot01$, the elliptic inequality for every ten minutes of mean solar time. A table of 1000 entries would give this quantity for equal increments of the argument corresponding to the movement in ten minutes from 0° to 90°. Pure copying of this table (backwards or forwards, with or without change of sign) would give all the quantities calculated with arguments wrong by the same constant Δg, say. It would then remain to calculate $22639''\cdot50\ \Delta g \cos g$, an operation not outside the limits of Crelle or Cotsworth's multiplication tables, if the correction be tabulated for the maximum value of Δg. The term in Δg^2 is not required. The modification is obvious, if the interval of the ephemeris is not to be 10 minutes, but say 12 hours; the table would then have to extend over 72 quadrants instead of one, and the number of entries in each quadrant will be one seventy-second part of the number (1,000) contemplated in the previous case.

There remains a special case to be alluded to, which is best illustrated by considering correction No. 23. The movement of the argument in one lunar day in this case is $63°\cdot995060$, and the successive convergents to $\frac{63°\cdot995060}{360°}$ are $\frac{1}{5}, \frac{1}{6}, \frac{2}{11}, \frac{3}{17}, \frac{8}{45}, \frac{283}{1592}$.

To form a table with $1592+45$ entries would be a labour altogether out of proportion to the end in view. A table, however, of $45+17$ entries is rather too small, as it would permit errors of $0''\cdot07$. Four tables of 45 entries each were therefore formed. The tables are lettered A, B, C, D. In each table the arguments are in arithmetical progression with a common difference of $63°\cdot995060$. The following extract from the value of the arguments

1 $= -a$	A 1 $= 180° - $D45	B 1 $= 180° + $D 1	C 1 $= 180° + $A 1
2 $= 180° + 3°\cdot85 - a$	A14 $= 180° - $D32	B32 $= 180° + $D32	C14 $= 180° + $A14
5 $= -0\cdot22 - a - 63°\cdot995$	A45 $= 180° - $D 1	B45 $= 180° + $D45	C45 $= 180° + $A45

shows that when one table has been constructed, the remaining tables follow by a change of sign, or a reversal of order, or both. It remains to explain the principles that determine the value of a, and the nature of the precepts.

When the computer reaches the end of any table, the precepts

usually direct him to recommence the same table. This, however, involves increasing the excess of the true argument over the argument of the table by $-0°·22$, and cannot be indefinitely repeated.

Occasionally, therefore, the precepts direct him to substitute

A14 for D1	C14 for B1
B2 for A1	D2 for C1

In other words, from time to time he has to change his table, always following the order of the alphabet, and having changed to go round and round the same table till the time comes for the next change.

Now, as the precepts are written, before settling the value of a it will be seen that the excess of the arguments D1, B1 over A14, C14 respectively is $3°·85-2a$; and the excess of the arguments A1, C1 over B2, D2 respectively is $2a$. Hence we are constantly increasing the excess of the true argument over the argument of the table by $-0°·22$, and we have the power, whenever we choose, of affecting this excess by a further increment, $3·85-2a$ or $2a$. It is now clear that we ought to take $a=0°·96$ so as to make them two quantities each equal to $1°·92$, when it will be seen that the power of adding $+1°·92$ at will enables us to keep the difference between the true argument and the tabular argument below the numerical value $0°·96$, corresponding to a maximum error $1''·03 \times 0°·96 - 0''·02$ in the use of the table.

Moreover, the method of tabulating the errors is convenient for the analysis of errors as well as applying corrections. Suppose, for example, it is required to analyse the errors for possible terms $g+g'$, or $g+g'$ with the addition of any long-period argument such as $\omega-\omega'$. We may take as an auxiliary angle θ, whose movement in one lunar day is $+14°·4$ and whose value is zero at each twenty-fifth day of each period of analysis. The ten sheets containing the 10×40 lunar days that go to one period of analysis are taken, and a cross is placed opposite columns 1 and 6, lines 25; columns 3 and 8, lines 20; columns 5 and 10, lines 15; columns 7 and 2, lines 10; columns 9 and 4, lines 5. The sheets are then placed so that the crosses fall in a row, whereupon the value of θ for all the observations in any row is the same. The sums are then taken down upon a convenient form for analysis into sine and cosine, and checked by the condition that the sum of the sums must be equal to the sum of all the errors for the whole period of analysis, a total that has been previously formed.

Now the movement of $g+g'-\omega+\omega'$ exceeds the movement of the auxiliary angle by $11°·1221$ in a period of analysis. Hence an error with argument $g+g'-\omega+\omega'$ will discover itself by a periodicity in a little less than 33 periods of analysis in the coefficients of sin θ and cos θ.

Further Analyses of Moon's Errors with Mean Elongation as argument, 1847–1901. By P. H. Cowell.

The present analyses are based upon the errors after thirteen corrections have been applied to Hansen's tabular places. These corrections are given in previous papers, where the periods of analysis are also defined.

In the annexed table the first column gives the number of the period of analysis. The next two columns are copied from last month's paper. The next three columns are fresh matter, and they are sufficiently distinguished by their headings. The meaning of the word "apparent" is that these are the corrections that are found by analysis upon the successive assumptions that each one alone exists. The subsequent columns, headed "resolved values," take account of the co-existence of the various separate errors. The relation between the "apparent" and "resolved" values is clearly dependent upon the distribution of the observations relatively to the age of the Moon. Relations were investigated, based upon the whole of the 48 periods of analysis (1847–1901). These relations should not strictly be applied to the individual periods, for any particular period may have an abnormal distribution of observations, but the error due to this cause has been allowed to coalesce with the accidental errors in the belief that they will not be so distributed as to sensibly affect the coefficients of deduced periodic corrections.

I now investigate the formulæ used. The notation is μ, δ_1, δ_2, Δ_1, Δ_2 stand respectively for the correction to semi-diameter, coefficients of sin D, sin 2D, cos D, cos 2D (resolved values). Accented letters denote apparent values. It will be seen that μ, δ_1, δ_2 fall into one group, Δ_1, Δ_2 into a second group, and that the two groups are kept separate.

First as to μ, δ_1, δ_2; in the December *Monthly Notices* I obtained from 5,647 observations normal equations:

$$5647\mu + 3788\delta_1 - 2327\delta_2 = \Sigma\pm\epsilon,$$
$$3788\mu + 3074\delta_1 - 1366\delta_2 = \Sigma\epsilon\,.\,\sin D,$$
$$-2327\mu - 1366\delta_1 + 2645\delta_2 = \Sigma\epsilon\,.\,\sin 2D,$$

ϵ being the error of any observation and the 48 times 400 lunar days being taken together.

For each of the present periods of analysis I put, n denoting the number of observations.

$$\mu' = \frac{1}{n}\Sigma\pm\epsilon$$

$$\delta'_1 = \frac{2}{n}\Sigma\epsilon \sin D.$$

$$\delta'_2 = \frac{5647}{2645n}\Sigma\epsilon \sin 2D.$$